Kuwait University	Math 101	Date:	May 30, 2006
Dept. of Math. & Comp. Sci.	Final Exam	Duration:	Two Hours

Calculators, mobile phones, pagers and all other mobile communication equipment are not allowed

Answer the following questions. Each question weighs 4 points.

1. Evaluate the following limits, if they exist:

   (a) $\lim\limits_{t\to 3} \dfrac{\frac{1}{t}-\frac{1}{3}}{t-3}$. (b) $\lim\limits_{x\to 0} \dfrac{\sin 7x}{3x^4-5x}$.

2. (a) State The Intermediate Value Theorem.

   (b) Let $f(x) = x^4 - 7x + 2$. Show that the equation $f(x) = 0$ has at least two distinct roots in the interval $[0, 2]$.

3. Determine whether $f(x) = x^3 + 1$ satisfies the hypotheses of The Mean Value Theorem on $[0, 3]$, and, if so, find the numbers c satisfying the conclusion of the theorem.

4. Given that $P = 2x^2 + 3y^2$ and $x - y = 10$, find x and y such that P is minimum.

5. Evaluate:

   (a) $\displaystyle\int \frac{\sin x}{(3+\cos x)^2}\,dx$. (b) $\displaystyle\int_0^2 \frac{5x^2}{\sqrt{2x^3+9}}\,dx$.

6. Show that if f is continuous, then
$$\int_0^1 f(x)\ dx = \int_0^1 f(1-x)\ dx\ .$$

7. Let $F(x) = 2\displaystyle\int_1^{3x} \frac{1}{t}\,dt - \int_2^{x^2} \frac{1}{t}\,dt$. Show that F is a constant function on $[1, \infty)$.

8. Find the average value, f_{av}, of $f(x) = \dfrac{3}{x^2}$ on $[-4, -1]$.

9. Find the area of the region bounded by the curves $y = x\sqrt{2x^2+1}$, $y = 0$, $x = 0$ and $x = 2$.

10. The region bounded by the curves $y = \sqrt{x-1}$, $y = 0$ and $x = 5$ is revolved about:

    (a) the line $y = -3$,

    (b) the line $x = 7$.

    Set up an integral that can be used to find the volume of the resulting solid in each case.

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Q1(a) $\lim_{t\to 3} \frac{\frac{1}{t}-\frac{1}{3}}{t-3} = \lim_{t\to 3}\left(\frac{3-t}{3t}\times\frac{1}{t-3}\right)$

$= \lim_{t\to 3} \frac{-\cancel{(t-3)}}{3t\cancel{(t-3)}} = -\frac{1}{9}$. Ans

Taslim

b) $\lim_{x\to 0} \frac{\sin 7x}{7x} \cdot \frac{7}{3x^3-5} = \frac{7}{-5} = -7/5$ Ans

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(Q2) a) Give Statement of I.M.V.T.

b) $f(x) = x^4 - 7x + 2$. let it has 3 roots say a, b, c

$\therefore f(a) = f(b) = f(c)$

Now by Rolle's Theorem there $c_1 \in (a,b)$ and $c_2 \in (b,c)$

Such that $f'(c_1) = f'(c_2) = 0$

Now there must be $l \in (c_1, c_2)$

Such that $f''(l) = 0$ but $f''(x) = 12x^2 \neq 0$ in (c_1, c_2)

again $f'(x) = 4x^3 - 7$. $\therefore x = (\frac{7}{4})^{\frac{1}{3}}$ $f'(x) = 0$ only for one value of x $\therefore$ we do not have c_2.

$\therefore$ Hence there are all at least two roots in (0, 2).

$f(0) = 2 > 0$
$f(1) = -4 < 0$
$f(2) = 4 > 0$
There is root in (0,1).
There is in (1,2)

Q3 $f(x) = x^3 + 1$ $f(3) = 28$ $f(0) = 1$

$f'(x) = 3x^2$

$\therefore \frac{f(b)-f(a)}{b-a} = f'(c) \Rightarrow \frac{28-1}{3} = 3c^2$

$\Rightarrow 9 = 3c^2 \Rightarrow c^2 = 3$ $c = \pm\sqrt{3}$

$\therefore c = \sqrt{3} \in (0, 3)$ Ans.

(Q4) $P = 2x^2 + 3y^2$ and $x - y = 10 \Rightarrow y = x - 10$

$\therefore P = 2x^2 + 3(x-10)^2$

$\frac{dP}{dx} = 4x + 6(x-10) = 0$

$10x = 60$ $x = 6$

$\frac{d^2P}{dx^2} = 4 + 6 = 10 > 0$ min

$x = 1$ gives min value $\therefore y = -4$

$(6, -4)$.

(Q5) (a) $\int \frac{\sin x}{(3+\cos x)^2}dx = +\frac{1}{3+\cos x} + C$. Ans

b) $\frac{5}{6}\int_0^2 \frac{6x^2}{\sqrt{2x^3+9}}dx = \frac{5}{6}\cdot 2\left|\sqrt{2x^3+9}\right|_0^2 = \frac{5}{3}(5-3) = \frac{10}{3}$ Ans

Q6 $\int_0^1 f(x)\,dx = \int_0^1 f(1-x)\,dx$

R.H.S let $1-x=u$ $\quad -dx = du \Rightarrow dx = -du$

when $x=0 \Rightarrow u=1$

$x=1 \Rightarrow u=0$

$\therefore \int_0^1 f(1-x)\,dx = -\int_1^0 f(u)\,du = \int_0^1 f(u)\,du$

Replace u by x $\therefore$ we get $\int_0^1 f(x)\,dx$ L.H.S

Q7 $F(x) = 2\int_1^{3x} \frac{1}{t}\,dt - \int_2^{x^2} \frac{1}{t}\,dt$.

$F'(x) = 2\cdot\frac{1}{\cancel{3}x}\cdot\cancel{3} - \frac{1}{x^2}\cdot 2x = \frac{2}{x} - \frac{2}{x} = 0$.

$F'(x)=0 \quad \therefore F(x) = C$ Ans

Q8) $f_{av} = \frac{1}{-1+4}\int_{-4}^{-1} \frac{3}{x^2}\,dx = \frac{\cancel{3}}{\cancel{3}}\left[-\frac{1}{x}\right]_{-4}^{-1} = -\left(-1+\frac{1}{4}\right) = +\frac{3}{4}$ Ans

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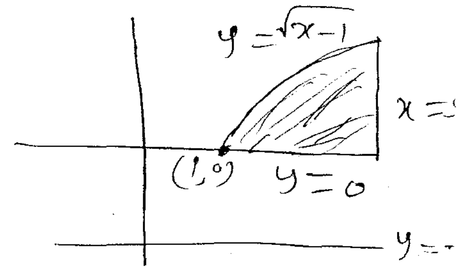
Q9 $A = \int_0^2 \sqrt{2x^2+1}\cdot x\,dx = \frac{1}{4}\cdot\frac{2}{3}\left.(2x^2+1)^{3/2}\right|_0^2$

$= \frac{1}{6}[27-1] = \frac{26}{6} = \frac{13}{3}$ Ans

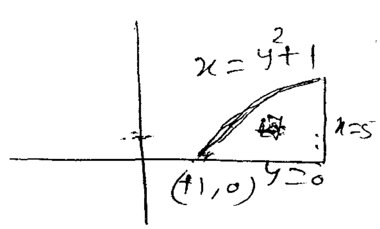
Q10) a)

$V = \pi\int_1^5 \left[(\sqrt{x-1}+3)^2 - 9\right]dx$.



b) $V = 2\pi\int_1^5 (7-x)\left[\sqrt{x-1} - 0\right]dx$

$V = 2\pi\int_1^5 (7-x)(\sqrt{x-1})\,dx$



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a) Shell $V = 2\pi\int_0^2 (y+3)\left[5-(y^2+1)\right]dy$

b) Washer $V = \pi\int_0^2 \left[\left[7-(y^2+1)\right]^2 - (7-5)^2\right]dy$.